BC Calc - Gold #5 solutions

① $\sqrt{x-2} \geq 0$

$x-2 \geq 0$

$x \geq 2$

D

② which is an odd function? i.e. $-f(x) = f(-x)$?

C

③ $g(x) = |\cos x - 1| = 2$

@ $x = \pi$

E

④ Period $= \dfrac{2\pi}{\frac{2\pi}{3}} = 3$

D

⑤ $f(g(x)) = \ln(9-x^2)$

$9-x^2 > 0$

$|x| < 3$

B

⑥ A

⑦ $x(x^2+4x+4) = 0$

$x(x+2)^2 = 0$

$x=0,\ x=-2$

B

⑧ $\lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}$

$= \dfrac{12}{4} = 3$ D

⑨ B look @ degrees

⑩ C

⑪ B

⑫ $\lim\limits_{x\to\infty} x\cdot\sin\left(\frac{1}{x}\right)$ $u = \frac{1}{x}$

$\lim\limits_{u\to 0} \dfrac{\sin u}{u} = 1$ E

⑬ $\lim\limits_{x\to 0} \dfrac{\cancel{x}(x-1)}{2\cancel{x}} = -\frac{1}{2}$

B

(14) $\lim_{h\to 0} \frac{\ln(e+h) - 1}{h}$

$\lim_{h\to 0} \frac{\ln(e+h) - \ln e}{h}$

$= f'(e)$

$f(x) = \ln x$

$f'(e) = \frac{1}{e}$ (B)

(15) $y = \frac{2-x}{3x+1}$

$y' = \frac{-(3x+1) - 3(2-x)}{(3x+1)^2}$

$= \frac{-7}{(3x+1)^2}$ (A)

(16) $y = (3-2x)^{1/2}$

$y' = \frac{1}{2}(3-2x)^{-1/2}(-2)$

$= \frac{-1}{\sqrt{3-2x}}$ (B)

(17) $y = \ln\left(\frac{e^x}{e^x - 1}\right)$

$y = \ln(e^x) - \ln(e^x - 1)$

$y = x - \ln(e^x - 1)$

$y' = 1 - \frac{1}{e^x - 1} \cdot e^x$

$= \frac{e^x - 1}{e^x - 1} - \frac{e^x}{e^x - 1} = \frac{-1}{e^x - 1} = \frac{1}{1 - e^x}$ (C)

(18) $y = \ln(\sec x + \tan x)$

$y' = \frac{1}{\sec x + \tan x} \cdot (\sec x \cdot \tan x + \sec^2 x)$

$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$

$= \sec x$ (A)

(19) $f(x) = \cos^2 x$

$f'(x) = -2\cos x \cdot \sin x = -2\sin 2x$

(C)

(20) $y = e^{-x}\cos 2x$

$y' = -e^{-x}\cos(2x) - e^{-x}\sin(2x) \cdot 2$

$= -e^{-x}(\cos(2x) + 2\sin(2x))$ (A)

(21) $x^3 - xy + y^3 = 1$

$3x^2 - y - x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$ (C)

(22) $x = \cos t$, $y = \cos 2t$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = -2\sin 2t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{-\sin t}$$

$$= \frac{-4\sin t \cdot \cos t}{-\sin t}$$

$$= 4\cos t$$

$$\frac{d^2y}{dt^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

$$= \frac{-4\sin t}{-\sin t}$$

$$= 4$$ (B)

(23) $x^2 + y^2 = 25$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-y - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,5)} = \frac{-5 + 0}{25} = -\frac{1}{5}$$ (D)

(24) $\sin x - \cos y - 2 = 0$

$$\cos x + \sin y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin y}$$

$$= -\cos x \cdot \csc y$$

(D)

(25) (E)

(26) $y = x \cdot \sin x$

$$\frac{dy}{dx} = \sin x + x \cos x$$

$$\left.\frac{dy}{dx}\right|_{\frac{\pi}{2}} = 1$$

$$y - \frac{\pi}{2} = x - \frac{\pi}{2}$$

$$y = x \quad \text{(E)}$$

(27) $y = \sqrt{2x+1}$

$$\frac{dy}{dx} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2$$

$$m = \frac{1}{\sqrt{2x+1}}$$

$$m_N = -\sqrt{2x+1} = -3$$

$$2x+1 = 9$$

$$x = 4$$
$$y = 3 \quad \text{(A)}$$

(28) $v(t) = 4t^3 - 18t^2 + 24t$

$$2t(2t^2 - 9t + 12) = 0$$

$$2t^2 - 9t + 12 \neq 0$$

$$2t = 0$$

$$t = 0$$

(B)

(29) $y = \dfrac{e^x}{x}$

$$\frac{dy}{dx} = \frac{x \cdot e^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$$

e^x	+		+		+
$x-1$	−		−		+
x^2	+		+		+
	(−)	0	(−)	1	(+)
	↘		↘		↗

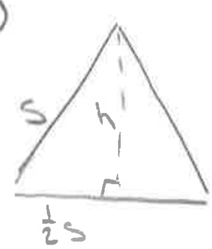
min at $x = 1$, $y(1) = e$ (D)

(30) (C)

(31) $f'(x) = \lim_{h \to 0} \dfrac{e^{x+h} - e^x}{h}$

$$f'(e) = \lim_{h \to 0} \frac{e^{e+h} - e^e}{h} \quad \text{(E)}$$

(32)



$$h^2 + \left(\tfrac{1}{2}s\right)^2 = s^2$$

$$h^2 = \frac{3}{4}s^2$$

$$h = \frac{\sqrt{3}}{2}s$$

$$A = \frac{1}{2}b \cdot h$$

$$= \frac{1}{2} \cdot s \cdot \frac{\sqrt{3}}{2}s$$

$$= \frac{\sqrt{3}}{4}s^2$$

$$\frac{dA}{ds} = \frac{\sqrt{3}}{2}s \qquad \left.\frac{dA}{ds}\right|_{s=2} = \sqrt{3}$$

(B)

(33)

$$h'(2) = \frac{1}{f'(1)}$$

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 4$$

$$h'(2) = \frac{1}{4} \quad \text{B}$$

(34) $y = 2e^{-x}$

$$x = 2e^{-y}$$

$$\frac{x}{2} = e^{-y}$$

$$\ln\left(\frac{x}{2}\right) = -y$$

$$y = \ln\left(\frac{2}{x}\right) \quad \text{A}$$

(35) $y = x^{\sin x}$

$$\ln y = \ln x^{\sin x}$$

$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$$

D

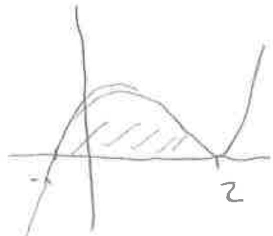
(36) $y = \tan^{-1}\left(\frac{x}{2}\right)$

$$y' = \frac{1}{2}\left(\frac{1}{1 + \frac{x^2}{4}}\right)$$

$$= \frac{1}{2 + \frac{x^2}{2}} = \frac{2}{4 + x^2}$$

E

(37)



$$A = \int_{-1}^{2} (x+1)(x-2)^2\,dx$$

$$A = \int_{-1}^{2} (x+1)(x^2 - 4x + 4)\,dx$$

$$= \int_{-1}^{2} (x^3 - 4x^2 + 4x + x^2 - 4x + 4)\,dx$$

$$= \int_{-1}^{2} (x^3 - 3x^2 + 4)\,dx$$

$$= \frac{1}{4}x^4 - x^3 + 4x\Big]_{-1}^{2} = (4 - \cancel{8} + \cancel{8}) - \left(\frac{1}{4} + 1 - 4\right)$$

$$= 6.75 \quad \text{E}$$

(38) $a(t) = 12t - 10$

$$v(t) = 6t^2 - 10t + c$$

$$3 = 0 - 0 + c$$

$$v(t) = 6t^2 - 10t + 3$$

$$s(t) = 2t^3 - 5t^2 + 3t + c$$

$$4 = 2 - 5 + 3 + c$$

$$c = 4$$

$$s(t) = 2t^3 - 5t^2 + 3t + 4$$

$$s(2) = 16 - 20 + 6 + 4$$

$$= 6 \quad \text{D}$$

(39) $f'(x) = 2(e^{4x^2}+5)$

(D)

(40) $\int_1^3 x(2x^2-5)^3\,dx$

$u = 2x^2-5$
$du = 4x\,dx$
$u(3) = 13$
$u(1) = -3$

$= \frac{1}{4}\int_{-3}^{13} u^3\,du$

(B)

(41) $\int (4-2t)^{1/2}\,dt$

$= -\frac{1}{2}\cdot\frac{2}{3}(4-2t)^{3/2} + C$

$= -\frac{1}{3}(4-2t)^{3/2} + C$

(A)

(42) $\int \frac{1-3y}{\sqrt{2y-3y^2}}\,dy$

$u = 2y-3y^2$
$du = 2-6y\,dy$
$= 2(1-3y)\,dy$

$= \frac{1}{2}\int u^{-1/2}\,du$

$= u^{1/2} + C$

$= (2y-3y^2)^{1/2} + C$

(E)

(43) $\int \frac{d\theta}{\cos^2 3\theta} = \int \sec^2 3\theta\,d\theta$

$= \frac{1}{3}\tan 3\theta + C$ (D)

(44) $\int \frac{\sin 2x}{\sqrt{1+\cos^2 x}}\,dx$

$u = 1+\cos^2 x$
$du = -2\cos x\cdot\sin x\,dx$
$= -\sin 2x\,dx$

$= -\int \frac{1}{\sqrt{u}}\,du$

$= -2u^{1/2} + C = -2\sqrt{1+\cos^2 x} + C$

(A)

(45) $\int \frac{e^x}{e^x-1}\,dx$

$u = e^x-1$
$du = e^x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln(e^x-1) + C$

(E)

(46) $\int (1-\frac{x}{2})^4\,dx$

$u = 1-\frac{1}{2}x$
$du = -\frac{1}{2}\,dx$

$= -2\int u^4\,du$

$= -\frac{2}{5}(1-\frac{1}{2}x)^5 + C$

(B)

47. $\int \frac{e^{2x}+1}{e^{2x}}\,dx$

$= \int \left(1 + \frac{1}{e^{2x}}\right) dx$

$= \int \left(1 + e^{-2x}\right) dx$

$= 1 - \frac{1}{2}e^{-2x} + C$

C

48. $\int 4 \cdot e^{1-2x}\,dx$

$4\left(-\frac{1}{2}e^{1-2x}\right) + C$

$-2e^{1-2x} + C$

A

49. $\frac{1}{\frac{\pi}{4} - \frac{\pi}{6}} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc^2 x\,dx$

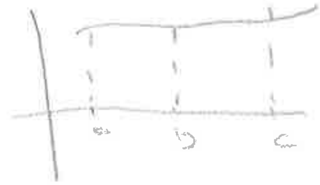
$\frac{12}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc^2 x\,dx$

$\frac{12}{\pi}\left[\cot x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$\frac{12}{\pi}\left[-1 + \sqrt{3}\right]$

C

50.



D

51. $\int_{-1}^{3} |x|\,dx$

using area

$= \frac{1}{2} + \frac{9}{2} = 5$ D

52. $\frac{d}{dt}\int_{0}^{t} \sqrt{x^3+1}\,dx$ FTC

$= \sqrt{t^3+1}$ A

53. $\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$

$= \tan x - x + C$ C

54. $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$

$\frac{1}{y}\,dy = \frac{1}{2\sqrt{x}}\,dx$

$\int \frac{1}{y}\,dy = \frac{1}{2}\int x^{-\frac{1}{2}}\,dx$

$\ln y = x^{1/2} + C$

$\ln 1 = 4^{1/2} + C$

$0 = 2 + C$

$\ln y = \sqrt{x} - 2$

$y = e^{\sqrt{x} - 2}$ E

56 $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$

$\frac{1}{\sin^2(\frac{\pi}{2}s)} ds = dt$

$\csc^2(\frac{\pi}{2}s)\, ds = dt$

$\int \csc^2(\frac{\pi}{2}s)\, ds = \int dt$

$-\frac{2}{\pi}\cot\left(\frac{\pi}{2}s\right) = t + C$

$-\frac{2}{\pi}\cot\left(\frac{\pi}{2}\right) = 0 + C$

$0 = C$

$t = -\frac{2}{\pi}\cot\left(\frac{\pi}{2}s\right)$

$t = -\frac{2}{\pi}\cot\left(\frac{\pi}{2}\cdot\frac{3}{2}\right)$

$= -\frac{2}{\pi}\cot\left(\frac{3\pi}{4}\right)$

$= -\frac{2}{\pi}(-1) = \frac{2}{\pi}$

D

57 $\int x \cdot f(x)\, dx$

$u = f(x) \qquad dv = x\, dx$

$du = f'(x)\, dx \quad v = \frac{1}{2}x^2$

$I = \frac{1}{2}x^2 \cdot f(x) - \int \frac{1}{2}x^2 \cdot f'(x)\, dx$

B

58 $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$dV = 4\pi(3)^2(.1) = 11.3$ A

59 $\int_0^1 \frac{1}{1+x^2}\, dx \approx$

$\frac{1}{2}(.2)\left[\frac{1}{1+0^2} + 2\left(\frac{1}{1+.2^2}\right) + 2\left(\frac{1}{1+.4^2}\right) + 2\left(\frac{1}{1+.6^2}\right) + 2\left(\frac{1}{1+.8^2}\right) + \frac{1}{1+1^2}\right]$

$= .1(1 + 1.923 + 1.724 + 1.471 + 1.213 + \frac{1}{2})$

$= .783$ A

60 $y = y_0 e^{kt}$

$\frac{1}{2} = e^{8k}$

$8k = \ln(.5)$

$k \approx -.0866$

$\frac{1}{3} = e^{kt}$

$t = \frac{\ln(\frac{1}{3})}{k} = 12.68\ \text{yr}$

E

(61) $a(t) = 4\pi \cos t$

$v(t) = 4\pi \sin t + C$

$0 = C$

$Av(f) = \frac{1}{\pi}\int_0^{\pi} 4\pi \sin t \, dt$

$= 4\int_0^{\pi} \sin t \, dt$

$= 4\left[-\cos t\right]_0^{\pi}$

$= 4[1+1] = 8$ (D)

(62) $f(x) = \sin x$, $f'(x) = \cos x$

$f'(c) = \dfrac{\sin(1.5) - \sin(1)}{1.5 - 1}$

$\cos c = .312$

$c = \cos^{-1}(.312)$

$= 1.253$ (D)

(63) $\int_0^k \sin x \, dx = \int_0^k x^2 \, dx$

$-\cos x \Big|_0^k = \frac{1}{3}x^3 \Big|_0^k$

$-\cos k + 1 = \dfrac{k^3}{3} - 0$

$\frac{1}{3}k^3 + \cos k - 1 = 0$

graph $k = 1.300$ (D)

(64)

$f(x) = \int_0^x (1 - 2\cos^3 t) \, dt$

$f'(x) = 1 - 2\cos^3 x$

$f'(x) > 0$ for

graph $.654 < x < 5.629$

(B)

(65) $\dfrac{dw}{dt} = k \cdot w$

$w = w_0 e^{kt}$

$\begin{cases} 36 = w_0 e^{2k}, & w_0 = \dfrac{36}{e^{2k}} \\ 84 = w_0 e^{7k} & w_0 = \dfrac{84}{e^{7k}} \end{cases}$

$36 = \dfrac{84}{e^{7k}} \cdot e^{2k}$

$36 = 84 \cdot e^{-5k}$, $k = .169459$

$36 = w_0 e^{2k}$, $w_0 = 25.651$

$\therefore w = w_0 e^{11k}$

$= 25.651 e^{11(.169459)}$

$= 165.448$ (E)

(66) $3x^2 - y^2 = 23$; $x = 4,\ y = 5$

$6x\frac{dx}{dt} - 2y\frac{dy}{dt} = 0$

$6(4)\frac{dx}{dt} - 2(5)(4) = 0$

$\frac{dx}{dt} = \frac{40}{12} = \frac{5}{3}$ units/sec

(67)

a) $f'(x) = \int f''(x)\,dx$

$= \int 24x - 18\,dx$

$f'(x) = 12x^2 - 18x + C$

$-6 = 12(1)^2 - 18(1) + C$

$0 = C$

$12x^2 - 18x = 0$

$6x(2x - 3) = 0$

$x = 0,\ x = \frac{3}{2}$

b) $f(x) = \int f'(x)$

$= \int 12x^2 - 18x\,dx$

$f(x) = 4x^3 - 9x^2 + C$

$0 = 4(2)^3 - 9(2)^2 + C$

$C = 4$

$f(x) = 4x^3 - 9x^2 + 4$

c) $av(f) = \int_1^3 f(x)\,dx$

$= 5$

(68) a) $f'(x) = 0$

$x = -7, -1, 4, 8$

b) Max when $f'(x)$ changes from pos to neg;

$x = -1,\ x = 8$

c) concave down; $f''(x) < 0$ or $f'(x)$ decreasing

$-3 < x < 2$ or $6 < x < 10$

(69) $v(t) = 2 - 3e^{-t}$

a) $a(t) = 3e^{-t}$

b) $s(t) = \int 2 - 3e^{-t}\,dt$

$= 2t + 3e^{-t} + C$

$0 = 2(0) + 3e^{-0} + C$

$-3 = C$

$s(t) = 2t + 3e^{-t} - 3$

$\Rightarrow$

(69) c) $v(t) = 0$

$2 - 3e^{-t} = 0$

$e^{-t} = \frac{2}{3}$

$t = -\ln(\frac{2}{3}) = \ln(\frac{3}{2})$

$t = .405$

d) $s(.405) \approx -.189$

(70) $P(t) = \dfrac{150}{1 + e^{4.3 - t}}$

* we haven't done logistic yet

(71) $\frac{dy}{dx} = 2y(x+1)$; $y(-2) = 2$

a) * estimate $y(0)$ using $\frac{dy}{dx} = 0.5$

$L(-1.5) = f(-2) + f'(-2)(.5)$

$= 2 + -4(.5) = 0$

$L(-1) = f(-1.5) + f'(-1.5)(.5)$

$= 0 + 0 = 0$

$L(-.5) = f(-1) + f'(-1)(.5)$

$= 0$

$L(0) = f(-.5) + f'(-.5)(.5)$

$= 0$

b) $\frac{dy}{dx} = 2y(x+1)$

$\frac{1}{y} dx = 2x + 2 \ dx$

$\ln y = x^2 + 2x + C$

$\ln 2 = (-2)^2 + 2(-2) + C$

$\ln 2 = 4 - 4 + C$

$C = \ln 2$

$y = e^{x^2 + 2x + \ln 2}$

$y = 2e^{x^2 + 2x}$

$y(0) = 2$